GCE A LEVEL MARKING SCHEME

SUMMER 2024

A LEVEL
FURTHER MATHEMATICS
UNIT 5 FURTHER STATISTICS B
1305U50-1

About this marking scheme

The purpose of this marking scheme is to provide teachers, learners, and other interested parties, with an understanding of the assessment criteria used to assess this specific assessment.

This marking scheme reflects the criteria by which this assessment was marked in a live series and was finalised following detailed discussion at an examiners' conference. A team of qualified examiners were trained specifically in the application of this marking scheme. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners. It may not be possible, or appropriate, to capture every variation that a candidate may present in their responses within this marking scheme. However, during the training conference, examiners were guided in using their professional judgement to credit alternative valid responses as instructed by the document, and through reviewing exemplar responses.

Without the benefit of participation in the examiners' conference, teachers, learners and other users, may have different views on certain matters of detail or interpretation. Therefore, it is strongly recommended that this marking scheme is used alongside other guidance, such as published exemplar materials or Guidance for Teaching. This marking scheme is final and will not be changed, unless in the event that a clear error is identified, as it reflects the criteria used to assess candidate responses during the live series.

WJEC GCE A LEVEL FURTHER MATHEMATICS

UNIT 5 FURTHER STATISTICS B

SUMMER 2024 MARK SCHEME

Qu.	Solution	Mark	Notes
1(a)	$\bar{x} = 29.95$	B1	
	Standard error = $\sqrt{\frac{0.6}{8}}$ $(= 0.2738\ldots)$	B1	SE^2 = $\frac{0.6}{8}$
	Use of $\bar{x} \pm z \times \text{SE}$	M1	FT their $\bar{x}$ and SE $\neq \sqrt{0.6}$ for M1A1 Allow z values for M1
	$=29.95 \pm 1.96 \times \sqrt{\frac{0.6}{8}}$	A1	1.96 or better
	$[29.41, 30.49]$	A1	cao 3sf or greater.
		[5]	
(b)	Two valid comments. e.g. We would have to use the t-distribution instead. e.g. Use 2.365 instead of 1.96. e.g. We would have to use t-tables. e.g. We would use the sample variance. e.g. We need to find s. e.g. Calculate an unbiased estimator of the variance.	E1 E1 **[2]**	Do not allow same reason twice.
(c)(i)	Valid explanation. e.g. The mean is outside the confidence interval so it may have been a different player. e.g. The average time is much higher than any of the other times recorded by the first player.	E1	
(ii)	Valid explanation. e.g. In part (a), the times refer to the first line drill of a practice session. Here it includes all the line drills in a practice session so the player may be tired and as a result have a higher mean time. e.g. The player may be returning from injury / may be injured. e.g. The player may be having an 'off' day.	E1 **[2]**	Do not condone there is a small probability that the player might score 35.6 at random. (The probability for this is in the order of magnitude $\times 10^{-95}$) Do not condone "5% chance it could be the player"
	Total for Question 1	**9**	

Qu.	Solution	Mark	Notes
2 (a)	H_0: The median daily caffeine intake per student from Country B who drinks coffee is 120mg H_1: The median daily caffeine intake per student from Country B who drinks coffee is greater than 120mg	B1	Let η be the median daily caffeine intake per student from Country B who drinks coffee. Both. $\eta = 120$, $\eta > 120$
	Diff x-120: 16, 29, 82, -10, -20, 60, 67, 18, 77, -5 Rank: 3, 6, 10, 2, 5, 7, 8, 4, 9, 1	M1 A1	Attempt to find differences All correct
	$w^+ = 3 + 6 + 10 + 7 + 8 + 4 + 9$ $w^+ = 47$ OR $w^- = 2 + 5 + 1$ $w^- = 8$	M1 A1	M1 Attempt at summing ranks. cao
	Upper CV = 44 OR Lower CV $(= \frac{1}{2} \times 10 \times (10+1) - 44)$ = 11 Since $47 > 44$ (OR $8 < 11$) there is sufficient evidence to reject H_0. It is reasonable to believe that the coffee-drinking students from country B drink more coffee than the students from country A.	B1 B1 E1 **[8]**	 FT their w^+/w^- and their CV cso
(b)	Valid comment on caffeine consumption. e.g. Caffeine intake may be from other sources so this should be taken into account. OR Valid comment on the exclusion of a sizable proportion of students. e.g. Whilst there may be evidence to suggest that the foreign students who drink coffee may drink more coffee it would be unwise to make this conclusion for the whole population because 1/3 of our sample did not drink any coffee. e.g Any conclusion made would be based on the coffee drinkers, which accounts for only 2/3 of the sample. e.g. Ignores students who have not drunk coffee on that particular day but maybe they usually do. OR Valid comment on experimental design. e.g. Small sample size. e.g. Could take a sample from both countries and compare. e.g. Data is only taken on one day.	E1 **[1]**	E0 for The time the caffeine intake was measured. E0 for The four students with zero caffeine intake may drink coffee late at night.
	Total for Question 2	**9**	

Qu.	Solution	Mark	Notes
3(a)	$\hat{p} = \frac{55}{80} = \frac{11}{16} = 0.6875$	B1	
	$\text{ESE} = \sqrt{\frac{0.6875 \times (1 - 0.6875)}{80}}$	M1	FT their $\hat{p}$ for M1A1
	$= 0.0518...$ or $\frac{\sqrt{11}}{64}$	A1	si
	$\hat{p} \pm z \times \text{ESE}$	M1	FT their $\hat{p}$ and ESE for M1A1
	$0.6875 \pm 1.6449 \times 0.0518...$	A1	1.645 or better
	$[0.602, 0.773]$	A1	cao
		[6]	
(b)	(90% of 50 =) 45	B1	
		[1]	
	Total for question 3	**7**	

Qu.	Solution	Mark	Notes
4(a)	$H_0: \mu_x - \mu_y = 0$ $H_1: \mu_x - \mu_y \neq 0$	B1 **[1]**	Both hypotheses. oe
(b)	Standard error of difference of means = $\sqrt{\frac{8^2}{40} + \frac{10^2}{40}}$	M1	
	$= 2.024(845673 \ldots)$	A1	si
	p-value $= 0.0692 \times 2$	A1	For 0.0692 Allow 0.06944 from tables
	$= 0.1384$	A1	Allow 0.1389
	Since $0.1384 > 0.05$ insufficient evidence to reject H_0.	m1	Accept other significance levels.
	There is not enough evidence to suggest that there is a difference in specific gravity of blood of cyclists and runners.	A1 **[6]**	cso
(c)	SE of difference in means = $\sqrt{\frac{8^2}{n} + \frac{10^2}{n}} \left(= \sqrt{\frac{164}{n}}\right)$ and difference of means $= 3$	M1	
	$\mathrm{P}\left(Z > \frac{3-0}{\sqrt{\frac{164}{n}}}\right) < 0.005$	M1	M1 Probability statement with "their mean and variance" and 0.005. May be implied by A1 oe Allow $=$ or $\leq$ (or $\geq$ for M1A1)
	$\frac{3-0}{\sqrt{\frac{164}{n}}} > 2.5758$	A1	Inequality ft "their mean and variance" and 2.576 or better.
	$\frac{3-0}{2.5758} > \sqrt{\frac{164}{n}}$		
	$n > 120.8998 \ldots$		$n < 120.8998 \ldots$ leading to $n = 121$ scores M1A0A1
	$n = 121$	A1 **[4]**	cao
	Total for question 4	**11**	

Qu.	Solution	Mark	Notes
5 (a) (i)	$\mathrm{E}(\bar{X}) = \mathrm{E}(X)$ $\mathrm{E}(X) = \int_0^\alpha x.\frac{3x^2}{\alpha^3}\mathrm{d}x$ $\mathrm{E}(X) = \left[\frac{3x^4}{4\alpha^3}\right]_0^\alpha$ $\mathrm{E}(X) = \frac{3\alpha}{4}$	M1 A1 A1	M1 for integrating $xf(x)\mathrm{d}x$ Limits not required Correct integration, limits required
	$\mathrm{E}(U) = \frac{4}{3}\mathrm{E}(\bar{X}) = \frac{4}{3}\times\frac{3\alpha}{4}$ $\mathrm{E}(U) = \alpha$ Therefore U is an unbiased estimator for α.	M1 A1 **[5]**	convincing
(ii)	$\mathrm{E}(X^2) = \int_0^\alpha x^2.\frac{3x^2}{\alpha^3}\mathrm{d}x = \left[\frac{3x^5}{5\alpha^3}\right]_0^\alpha$ $\mathrm{E}(X^2) = \frac{3\alpha^2}{5}$ $\mathrm{Var}(X) = \frac{3\alpha^2}{5} - \left(\frac{3\alpha}{4}\right)^2$ $\mathrm{Var}(X) = \frac{3\alpha^2}{80}$ $\mathrm{Var}(U) = \left(\frac{4}{3}\right)^2\mathrm{Var}(\bar{X})$ $\mathrm{Var}(U) = \frac{16}{9}\times\frac{3\alpha^2}{80n}$ $\mathrm{Var}(U) = \frac{\alpha^2}{15n}$ $\mathrm{SE} = \sqrt{\frac{\alpha^2}{15n}}$ therefore $n = 15$	M1 A1 M1 A1 M1 M1 A1 M1 A1 **[9]**	Attempt to integrate Limits not required FT their $\mathrm{E}(X)$ and $\mathrm{E}(X^2)$ cao Use of $a^2\mathrm{Var}(\bar{X})$. FT their $\mathrm{Var}(X)$ cao FT their $\mathrm{Var}(U)$ cao
(b)(i)	$\mathrm{Var}(V) = 4^2\mathrm{Var}(\bar{X}_1) + \left(\frac{8}{3}\right)^2\mathrm{Var}(\bar{X}_2)$ $\mathrm{Var}(V) = 16\times\frac{3\alpha^2}{80n} + \frac{64}{9}\times\frac{3\alpha^2}{80n}$ $\mathrm{Var}(V) = \frac{13\alpha^2}{15n}$ $\frac{\mathrm{Var}(U)}{\mathrm{Var}(V)} = \frac{\alpha^2}{15n}\times\frac{15n}{13\alpha^2} = \frac{1}{13}$ **ag***	M1 m1 A1 B1 **[4]**	Use of $a^2\mathrm{Var}(\bar{X})$. FT their $\mathrm{Var}(\bar{X})$ cao Convincing
(b)(ii)	Since $\frac{\mathrm{Var}(U)}{\mathrm{Var}(V)} < 1$, U is the better estimator.	E1 **[1]**	
	Total for Question 5	**19**	

Qu.	Solution	Mark	Notes
6	H_0: The median numbers of words memorised by Group A and Group B are the same. H_1: The median number of words memorised by Group B is more than the median number of words memorised by Group A.	B1	Accept $H_0: \eta_A = \eta_B$ $H_1: \eta_B > \eta_A$ Also accept $H_1: \eta_A < \eta_B$
	Use of the formula $U = \sum\sum z_{ij}$	M1	Attempt to use
	$U = 2 + 9 + 6 + 7 + 9 + 6 + 6 + 6 + 6 + 6 + 5 + 7$ OR $U = 1 + 1 + 10 + 2 + 0 + 0 + 1 + 10 + 8$		Accept ranks with opposite signs ALTERNATIVE METHOD $t_A =$ sum of ranks for $A = 111$ $U_A = 111 - \frac{13\times12}{2} = 33$ $B =$ sum of ranks for $B = 120$ $U_A = 120 - \frac{9\times10}{2} = 75$
	$U = 75$ OR $U = 33$	A1	
	Upper critical value is 78 OR Lower CV is 30	B1	
	75 < 78 OR 33 >30, there is insufficient evidence to reject H_0.	m1	
	There is insufficient evidence to suggest that memorising words that are different in meaning is easier than memorising words that are similar in meaning. There is insufficient evidence to suggest that group B are better at memorising words than group A.	A1 **[6]**	cso
	Total for Question 6	**6**	

Qu.	Solution	Mark	Notes
7(a)	$\mathrm{P}(W < 19) = 0.1$ $\mathrm{P}(W < 19) = P\left(Z < \frac{19-\mu}{0.6}\right)$ $\frac{19-\mu}{0.6} = -1.282$ $\mu = 19.8$	 M1 A1 A1 **[3]**	 M1 for attempt to standardise with 19 and 0.6 oe
(b)	$\overline{X} \sim \mathrm{N}\left(20.1, \frac{1.2^2}{8}\right)$ $\mathrm{P}(\overline{X} > 20) = 0.593\ldots$ ***ALTERNATIVE SOLUTION*** Let $L = X + X + X + X + X + X + X + X$ $L \sim \mathrm{N}(160.8, 8 \times 1.2^2)$ $\mathrm{P}(L > 160) = 0.593\ldots$	B1 M1A1 (B1) (M1) (A1) **[3]**	si M1 for $z = \frac{20-20.1}{\sqrt{0.18}}$ or $z = -0.24$ A1 for 0.59483 FT for M1 only provided $\sigma \neq 1.2$ M1 for $z = \frac{160-160.8}{\sqrt{11.52}}$ or $z = -0.24$ A1 for 0.59483
(c) (i)	Let $T = Y + Y + Y - X - X - X$ $\mathrm{E}(T) = 6.3$ $\mathrm{Var}(T) = 3 \times 1.5^2 + 3 \times 1.2^2$ $\mathrm{Var}(T) = 11.07$ $\mathrm{P(move\ the\ einkorn)} = \mathrm{P}(T > 0) = 0.97085$	 B1 M1 A1 A1	 cao
(ii)	$\mathrm{P(move\ the\ corn)} = \mathrm{P}(T < 0) = 1 - \mathrm{P}(T > 0)$ $\mathrm{P(move\ the\ corn)} = \mathrm{P}(T < 0) = 0.02915$	 B1 **[5]**	 FT $1 - (i)$
(d) (i)	Let $U = X - 3E$ $\mathrm{P}(U > 0) = 0.35208$ $\mathrm{E}(U) = -11.4$ $\mathrm{P}(Z > \frac{0--11.4}{\sigma_U}) = 0.35208$ $\frac{11.4}{\sigma_U} = 0.37971$ $\sigma_U = 30.022912\ldots\ldots$ $\mathrm{Var}(U) = \mathrm{Var}(X) + 9\mathrm{Var}(E)$ $901.37525 = 1.2^2 + 9\mathrm{Var}(E)$ $\sigma = 10$	 B1 B1 B1 M1 A1 M1 A1	 B1 for use of 0.35208, si B1 for $\mathrm{E}(U)$ B1 for ± 0.37971 M1 for equation si M1 for use of variance formula.

Qu.	Solution	Mark	Notes
7(d) (i)	***ALTERNATIVE SOLUTION*** $\mathrm{P}(X > 3E) = 0.35208$ $X - 3E \sim N(20.1 - 3 \times 10.5, 1.2^2 + 9\sigma^2)$ $X - 3E \sim N(-11.4, 1.2^2 + 9\sigma^2)$ $\mathrm{P}(X > 3E) > 0$ $\mathrm{P}\left(Z > \frac{0 - -11.4}{\sqrt{1.2^2 + 9\sigma^2}}\right) = 0.35208$ OR $\mathrm{P}\left(Z < \frac{0 - -11.4}{\sqrt{1.2^2 + 9\sigma^2}}\right) = 0.64792$ $\frac{11.4}{\sqrt{1.2^2 + 9\sigma^2}} = 0.3797$ $11.4^2 = 0.3797^2 \times (1.2^2 + 9\sigma^2)$ $\sigma^2 = \frac{129.96 - 0.20761}{0.3797^2 \times 9}$ $\sigma^2 = 99.99808023$ $\sigma = 10$	(B1) (B1) (M1) (B1) (M1) (A1) (A1)	B1 for use of 0.35208, si B1 for -11.4 M1 for Variance. B1 for ± 0.3797 M1 for equation A1 rearranging to get $\sigma^2 = k$
7(d) (ii)	Valid comment e.g. (The value of σ is far too big.) It gives a high probability that the mass would be negative, which is obviously impossible. e.g. (The value for σ is far too big.) It's almost as big as the mean.	E1 **[8]**	Must give reason more than just "σ is too big".
	Total for Question 7	**19**	